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Role of disorder in inducing a BCS–BEC crossover

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Abstract

The role of random onsite and hopping disorder in inducing a Bardeen–Cooper–Schrieffer superconductor to Bose–Einstein condensate (BCS–BEC) crossover is investigated using the Bogoliubov–de Gennes (BdG) method on a negative-U Hubbard model in two dimensions. In the strong disorder limit, the BdG state becomes highly inhomogeneous with the formation of superconducting '*droplets*' which are characterized by large pairing amplitudes. The different physical properties calculated for the inhomogeneous state differ significantly from the pure system at all values of electron densities, while we show that a crossover to a BEC state, if at all, is possible at low electron densities. The random onsite disorder supports such a crossover scenario, which is confirmed by observing the chemical potential to slip below the noninteracting band (Leggett criterion). The hopping disorder, however, renders a delocalized phase which allows for overlapping between the pairs, thus preventing the formation of a 'local pair' phase. Also the increase in bandwidth induced by hopping disorder always accommodates the chemical potential inside the band, and hence rules out a crossover phenomenon. However, another variant of the off-diagonal inhomogeneity, i.e. an anisotropic hopping, offers a pro-crossover scenario in the large anisotropy limit.

1. Introduction

The problem of crossover from a Bardeen–Cooper–Schrieffer superconductor (BCS) characterized by largely overlapping Cooper pairs to a Bose–Einstein condensate (BEC) of tightly bound, short ranged pairs has invoked enormous excitement among physicists in recent times. The initial attempts on the BCS–BEC crossover phenomenon were made by Eagles [1] and later by Leggett [2]. Using a variational approach, they had shown that the BCS ground state wavefunction is capable of describing a smooth transition from a BCS ground state to a BEC of diatomic molecules by a controlled tuning of the strength of the attractive interaction at $T = 0$. Further work on the subject of crossover, done by Nozieres and Schmitt–Rink (NSR) [3], was to extend the problem to finite temperatures and in models with discrete symmetries, i.e. lattices. They determined the transition temperature, T_c that separates the superconducting and normal phases, via the Thouless criterion. NSR showed that T_c continuously evolves as a function of attractive interaction between the fermions.

Since the discovery of short coherence length superconductors (e.g. the cuprates), which are believed to occur in between large coherence length conventional superconductors (BCS) and a phase with extremely short (of the order of one lattice spacing) pairing correlations, the crossover phenomenon has received renewed attention. Attempts were made to interpolate between the two extreme limits so as to visit the physics that is operative for these superconductors. In this regard, Randeria and coworkers [4] initiated formal studies where a dilute gas of fermions in two dimensions interacting via a given two-body potential is shown to display a smooth crossover from a BCS to a BEC phase. Subsequently a great deal of work using variational techniques [4–6], functional integral methods [7], the diagrammatic approach [8–13], numerical studies such as quantum Monte Carlo [14, 15] and dynamical mean field theory [16] followed which have extensively investigated the normal and superconducting states (both above and below T_c). Some of these works have systematically gone beyond the standard mean field approximation to incorporate thermal and quantum fluctuations.

There is a growing consensus that the crossover scenario may provide us with a route to understand unusual features exhibited by the unconventional superconductors. Based on the μSR and magnetic field penetration data, Uemura *et al* [17] suggested that the unconventional or the 'exotic' superconductors belong to a separate class of materials

characterized by high-T_c, short coherence length ξ and low superfluid density, which are *unlike* their conventional counterparts. Further, the short ξ found in these materials, generate a scenario which is akin to a system consisting of local bosons. Finally, the boson condensation temperature, T_B, at which the noninteracting Bose gas condenses (T_B is analogous to T^* at which local pairs form in the underdoped phase of cuprates), is almost one order of magnitude greater than the superconducting transition temperature T_c in these systems. The results encourage a generalization of the BCS theory to be more suitable to the situation, rather than pursuing a search for more exotic theories [18–20]. In fact the coherence length, ξ (or its dimensionless variant $k_F\xi$, k_F being the Fermi wavevector) is an appropriate quantity that tracks the crossover phenomenon [21], such that at $k_F\xi \simeq 2\pi$, the system becomes unstable against bosonization and the concept of Fermi surface gets wiped out. Further, a meaningful connection to the Uemura plot is obtained as the unconventional materials are found to lie near the $k_F\xi \simeq 2\pi$ line in the plot and hence are closer to Fermi surface instability.

The theoretical endeavours gathered momentum with the experimental realization of a BCS–BEC crossover occurring in a dilute gas of Fermi atoms subjected to a magnetic trap, where the pairing potential (scattering length) is manipulated by a Feshbach resonance [22]. An extensive list of references on the subject of crossover, its relevance to the cuprates, the experimental achievements and the theoretical triumph may be found in [23] and [24].

So far we have discussed the evolution from BCS to a BEC as the coupling strength associated with an effective fermionic potential is enhanced. We next focus on the significance of the density of charge carriers in causing a crossover. In fact, the combined effects of density and interparticle potential [25] have been investigated and yield important conclusions such as the robustness of the crossover scenario for all densities in the case of s-wave pairing; however, the same is not true for d-wave pairing at large densities. The reason being, at large densities, there is a substantial overlap between the spatially extended (d-wave) pairs even for strong attractive interactions. An elaborate discussion on density induced crossover in the presence of different types of fermionic interaction potentials has also been done by Andrenacci *et al* [26]. They have shown that the absence of a threshold (for the formation of a two-body bound state) in d-wave systems and the existence of a finite range of potential favour density driven crossover in two-dimensional lattice systems. In addition, they have found out the essential criterion for the presence of crossover (as a function of density) in the continuum case.

We turn towards yet another quantity that is capable of driving a system through crossover. The role of structural disorder in inducing a smooth evolution from a BCS superconductor to a BEC superfluid was put forward earlier [28]. More recently, a strongly disordered attractive Hubbard model with infinite range hopping (where mean field theory is exact) is shown to emulate a smooth BCS–BEC crossover as the range of hopping is varied [29]. Further, the ground state properties of a superfluid Fermi gas are studied across a BCS–BEC crossover in the presence of random disorder at $T = 0$ [30]. The main observation in [34] is that the superfluid order parameter shows a non-monotonic behaviour as a function of interaction strength with a pronounced dip near the crossover regime. The renormalization effects are more pronounced as one moves towards the BEC limit.

The various works listed above do not put emphasis on evolution from a BCS to a BEC as a function of the strength of disorder alone. The investigation of the effect of disorder on the superconducting properties of dirty metals has a long history dating back almost to the advent of BCS theory [31]. Renewed efforts during the 1980s [32–34] have revealed that in the strong disorder regime, the pairs lose their large spatial extension and become localized. A good review on the subject may be found in [39]. From the ongoing discussion, it is clear that disorder crucially affects superconductivity, however, at low densities, as we shall show later, it induces a crossover from a homogeneous (BCS) phase to a BEC superfluid.

Numerics, to investigate disorder effects on superconductivity, were used extensively [36–40]. Since the main objective was to study how superconducting properties respond to disorder and because the superconducting correlations are strongest at large electron densities, these studies were mostly restricted to large densities (and weak coupling). In any case, the main observation was that the spectral gap persists in the presence of disorder, as is evident from the formation of superconducting islands, characterized by large pairing amplitudes and separated by regions which are insulating in nature. As the strength of disorder is enhanced, the islands shrink, making room for the intervening insulating seas. At small electron densities, the islands (or what we call '*droplets*' here) are more localized and thus bear fingerprints of a BEC phase which has very short and local pairs. However, we should be cautious in issuing a statement such as above, since this, if at all, will be true only in the mean field sense. In the presence of strong disorder, quantum fluctuations, the importance of which are underscored in the literature [37, 39], will play a crucial role.

In this paper, we study superconductivity in the presence of random onsite and hopping disorder using Bogoliubov–de Gennes (BdG) method [41] on a negative-U Hubbard model in two dimensions. BdG renders an appropriate treatment to the spatial inhomogeneity of the local pairing amplitude induced by disorder. Hopping anisotropy is included for comparison with our earlier studies on the subject [43]. The main objective is to obtain a smooth evolution from a BCS ground state to a 'local pair' phase whose properties bear much in common with the BEC. However, such a crossover is only possible at low densities as will be explained below and the crossover scenario is confirmed by calculating the chemical potential which slips below the noninteracting band, thus tending to the limit of binding energy of a pair (Leggett criterion).

In the next section, the model Hamiltonian and a brief description of the BdG method are presented. The third section deals with a detailed description on the subject of crossover in disordered superconductors, where we discuss different candidates, such as random onsite and hopping disorder, and in addition study the case of anisotropic hopping. A summary of the results and their implications are presented in section 4.

2. The model

We consider an attractive Hubbard model on a two-dimensional square lattice,

$$H = -\sum_{\langle ij\rangle,\sigma}(t+\delta t_{ij})(c^{\dagger}_{i\sigma}c_{j\sigma}+\text{h.c.}) - |U|\sum_i n_{i\uparrow}n_{i\downarrow}+\sum_{i,\sigma}(V_i-\mu)n_{i\sigma}. \quad (1)$$

Here t is the transfer integral, $c^{\dagger}_{i\sigma}(c_{i\sigma})$ is the creation (destruction) operator for an electron with spin σ at a site $\mathbf{r}_i$, $|U|$ is the magnitude of onsite interaction, $n_{i\sigma}=c^{\dagger}_{i\sigma}c_{i\sigma}$ and μ denotes the chemical potential. To model random onsite disorder, we let $\delta t_{ij}=0$ for all i and j and V_i be chosen randomly from a Gaussian distribution of the form,

$$P[V_i(\sigma)]=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left[-\frac{V_i^2}{2\sigma^2}\right]. \quad (2)$$

In a similar fashion, hopping disorder is modelled by letting $V_i=0$ for all i and δt_{ij} to be chosen from a Gaussian distribution, which may be obtained as one replaces V_i by δt_{ij} in equation (2). Here σ denotes the width of the distribution and parametrizes the strength of disorder in our computation, i.e. larger σ implies stronger disorder. The third kind of inhomogeneity, i.e. hopping anisotropy, which is a case of correlated disorder, is modelled trivially by choosing $\delta t_{ij}=V_i=0$ and the hopping is t when i and j are neighbours in the x-direction, while it is rt when j is the y-neighbour of i, with r the anisotropy parameter ($0<r\leqslant 1$).

To solve equation (1), we resort to the mean field decoupling of the interacting term that yields the local pairing amplitudes and local density as,

$$\Delta(\mathbf{r}_i)=-|U|\langle c_{i\downarrow}c_{i\uparrow}\rangle \qquad \text{and} \qquad \langle n_{i\sigma}\rangle=\langle c^{\dagger}_{i\sigma}c_{i\sigma}\rangle. \quad (3)$$

The effective Hamiltonian is given by,

$$H_{\text{eff}}=-\sum_{\langle ij\rangle,\sigma}(t+\delta t_{ij})(c^{\dagger}_{i\sigma}c_{j\sigma}+\text{H.c.})+\sum_{i,\sigma}(V_i-\tilde{\mu}_i)n_{i\sigma} + \sum_i[\Delta(\mathbf{r}_i)c^{\dagger}_{i\uparrow}c^{\dagger}_{i\downarrow}-\Delta^*(\mathbf{r}_i)c_{i\uparrow}c_{i\downarrow}] \quad (4)$$

where $\tilde{\mu}_i=\mu+|U|\langle n_i\rangle/2$ and $\langle n_i\rangle=\sum_\sigma\langle n_{i\sigma}\rangle$. The following transformations are used to diagonalize equation (4) [39],

$$c_{i\uparrow}=\sum_n[\gamma_{n\uparrow}u_n(\mathbf{r}_i)-\gamma^{\dagger}_{n\downarrow}v^*_n(\mathbf{r}_i)],\qquad c_{i\downarrow}=\sum_n[\gamma_{n\downarrow}u_n(\mathbf{r}_i)+\gamma^{\dagger}_{n\uparrow}v^*_n(\mathbf{r}_i)] \quad (5)$$

where γ_n and $\gamma^{\dagger}_n$ are the quasiparticle operators, $u_n(\mathbf{r}_i)$ and $v_n(\mathbf{r}_i)$ are the BdG eigenvectors satisfying $\sum_n[u_n^2(\mathbf{r}_i)+v_n^2(\mathbf{r}_i)]=1$ for all $\mathbf{r}_i$. In terms of these amplitudes, equation (4) is written as,

$$\begin{pmatrix}\hat{K} & \hat{\Delta}\\ \hat{\Delta}^* & -\hat{K}^*\end{pmatrix}\begin{pmatrix}u_n(\mathbf{r}_i)\\ v_n(\mathbf{r}_i)\end{pmatrix}=E_n\begin{pmatrix}u_n(\mathbf{r}_i)\\ v_n(\mathbf{r}_i)\end{pmatrix} \quad (6)$$

where $\hat{K}u_n(\mathbf{r}_i)=-t\sum_\delta u_n(\mathbf{r}_i+\hat{\delta})+(V_i-\tilde{\mu}_i)u_n(\mathbf{r}_i)$, with $\hat{\boldsymbol{\delta}}=\pm\hat{x},\pm\hat{y}$ and $\hat{\Delta}u_n(\mathbf{r}_i)=\Delta(\mathbf{r}_i)u_n(\mathbf{r}_i)$ and similarly for $v_n(\mathbf{r}_i)$.

Hence we calculate the local pairing amplitudes and number density in terms of $u_n(\mathbf{r}_i)$ and $v_n(\mathbf{r}_i)$ using,

$$\Delta(\mathbf{r}_i)=|U|\sum_n u_n(\mathbf{r}_i)v^*_n(\mathbf{r}_i) \qquad \text{and} \qquad \langle n_i\rangle=2\sum_n|v_n(\mathbf{r}_i)|^2. \quad (7)$$

The usual procedure for obtaining $\Delta(\mathbf{r}_i)$ and $\langle n_i\rangle$ self-consistently consists of making initial guesses for $\Delta(\mathbf{r}_i)$ and the renormalized chemical potential $\tilde{\mu}_i$ for all $\mathbf{r}_i$, diagonalize equation (6) for the eigensolutions E_n and $(u_n(\mathbf{r}_i),v_n(\mathbf{r}_i))$ recompute $\Delta(\mathbf{r}_i)$ and $\langle n_i\rangle$ using equation (7). The process is iterated until self-consistency is achieved for these quantities at each site.

Equation (7) can be generalized to finite temperatures, however, an attempt to interpolate between BCS (at small $|U|$) and BEC (at large $|U|$) at finite temperatures, may be inappropriate as the mean field transition temperature T_c in the BEC limit is found to be proportional to $|U|$, whereas the actual scale is found to be that of J ($\sim t^2/|U|$) [16]. In this paper, we have restricted ourselves to $T=0$ (except for a brief discussion on figure 4) and kept $|U|$ unaltered (an interpolation from BCS to BEC is attempted by tuning the disorder), yet quantum fluctuations, as mentioned earlier, will be there.

At the outset we discuss the size dependence issues related to our computation. We have obtained ground state energy, E_{gs} and superfluid stiffness, D_{s} for $|U|=1.5t$ and $n=0.1$ and lattice sizes up to 24×24. (See figure 1). Energies are among the quantities which are not too sensitive to system size. However, D_{s} (defined in equation (8)) is denoted in terms of correlation functions, and is likely to display dependence on system size. Both these quantities plotted corresponding to the pure case, as seen in figure 1, do not show any appreciable dependence on size, for sizes greater than 12×12 and since we are primarily interested in qualitative behaviour of physical quantities that emphasize the crossover phenomenon, we decided to carry out our computation on a 12×12 lattice.

Next we comment on the choice of physical parameters. We perform studies for a few choices of the Hubbard interaction parameter U and electron density n. The choice of U is expectedly small, as our starting point is a weak coupling superconductor (BCS). As for the density, figure 2 shows that the chemical potential slips below the noninteracting band only at small n (for moderate values of U). It is important to note that the chemical potential μ (in fact, a scaled value, namely $\mu'=\mu/4t$ is useful for discussion) does not fall below the band edge ($\mu'=-1$) for $|U|$ less than $\sim 5.5t$ even for density as low as 0.1. In the next section we shall show that disorder can induce a crossover at much smaller values of U, i.e. $|U|=1.5t$ at the same density ($n=0.1$). At large densities, namely $n\simeq 0.8$, as is transparent from figure 2, there is no crossing of μ' below -1 even at reasonably large coupling, thereby making it impossible for disorder to induce a crossover at weak coupling. Thus we have chosen $|U|=1.5t$,

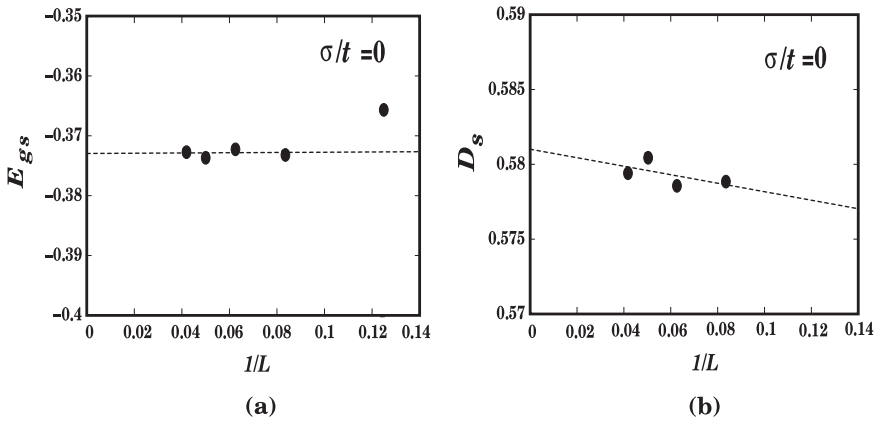

Figure 1. The size dependencies of the ground state energy E_{gs} (a) and superfluid stiffness D_s (b) are shown up to lattice sizes 24×24 for the pure case. A best fit line drawn shows no appreciable size dependencies for sizes 12×12 onwards. As mentioned in the text, all figures (except for figure 2) are plotted corresponding to $|U| = 1.5t$ and $n = 0.1$ and not repeated hereafter.

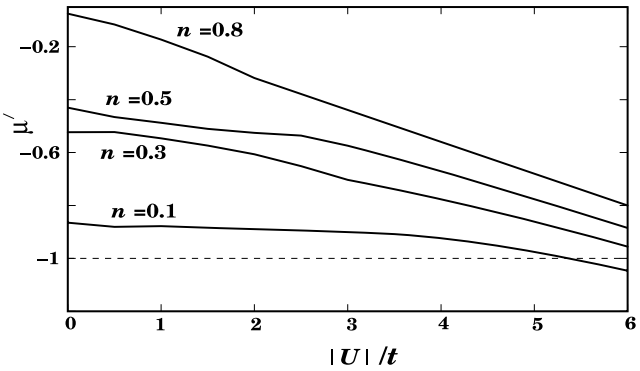

Figure 2. The (scaled) chemical potential μ' ($= \mu/4t$, $4t$ being the noninteracting bandwidth) is shown as a function of $|U|$. The slipping below the band occurs for low densities and at best moderate couplings, i.e. $|U| \sim 5.5t$ for $n = 0.1$.

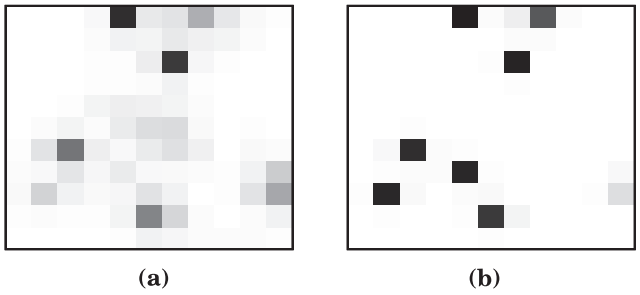

Figure 3. Greyscale plots of spatial charge distribution $\langle n_i \rangle$ ($= 2\sum_n v_n^2(\mathbf{r}_i)$) for disorder strengths $\sigma/t = 1$ and 3. The localized nature of the charge densities is to be noted for larger disorder.

$n = 0.1$ and all the plots (except for figure 2) are generated for these values of the parameters. Further, we have investigated disorder strengths till $\sigma/t = 3$ and all the results for various physical quantities that we obtained are averaged over ten different disorder configurations.

3. Results

In the subsequent discussion, we present results of various physical quantities that are calculated to gain insight into the effect of disorder on superconductivity and on the issue of BCS–BEC crossover.

3.1. Onsite disorder

We plot the spatial distribution of electron occupancies $\langle n_i \rangle$ ($= 2\sum_n v_n^2(\mathbf{r}_i)$ in figure 3 for two different disorder strengths, namely $\sigma/t = 1$ and 3. The plot corresponding to stronger disorder shows localized electron occupancies and thus is supportive of a phase comprising of short and local pairs, reminiscent of a BEC phase. Such plots are also available for higher densities (not shown here), where the electron occupancies show considerable overlap. Further, plots for pair amplitudes $\Delta(\mathbf{r}_i)$ are also available, they are discussed in [41, 43] and carry no extra information than the $\langle n_i \rangle$ distribution mentioned above, hence are not included here.

Evolution from a BCS state to a BEC is merely a suspicion at this moment and we shall get back to the issue soon. Meanwhile the renormalization of various quantities as the disorder strength is varied is certainly a matter of interest. Ghosal *et al* [39] have investigated many of the important quantities in detail. We shall not repeat them (albeit their calculations are done at high densities), but only consider a few of them which are relevant to us.

In what follows, we show a gradual emergence of unconventional features in the system with disorder that merit discussion. We begin with the notion of off-diagonal long range order (ODLRO). The ODLRO order parameter Δ_{op} is defined by the long distance behaviour of the disorder averaged correlation $\langle c_{i\uparrow}^{\dagger} c_{i\downarrow}^{\dagger} c_{j\downarrow} c_{j\uparrow} \rangle \rightarrow \Delta_{OP}^2/|U|^2$ for $|\mathbf{r}_i - \mathbf{r}_j| \rightarrow \infty$ [44]. We plot the temperature dependence of Δ_{op} and the double occupancy n_d ($=\langle n_{i\uparrow} n_{i\downarrow} \rangle$) between the BdG states with and without disorder (figure 4) [14]. The temperature at which they vanish (or become negligible) is the same for the pure system, while it differs considerably for $\sigma/t = 1$. The result is important in the sense that it shows pair formation (indicated by n_d) and condensation (indicated by Δ_{op}) are same for the

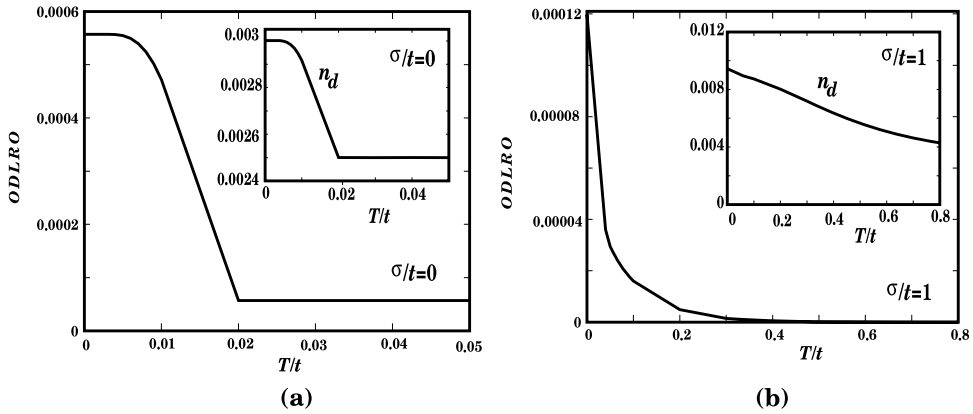

Figure 4. The off-diagonal long range order ODLRO and double occupancy $n_{\rm d}$ are shown as a function of temperature T for $\sigma/t = 0$ (a) and 1 (b). The pure case shows a unique temperature scale for them to become negligible, while they occur at considerably different temperatures for the disordered case.

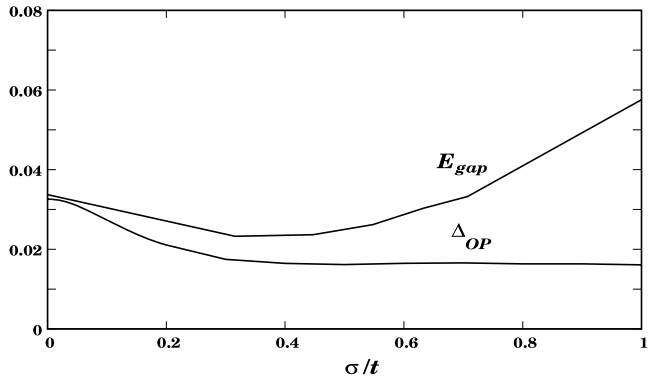

Figure 5. The spectral gap, $E_{\rm gap}$ and the ODLRO order parameter, $\Delta_{\rm OP}$ (defined in the text) are shown as a function of disorder. Note that they coincide for small σ/t but progressively differ with increasing disorder.

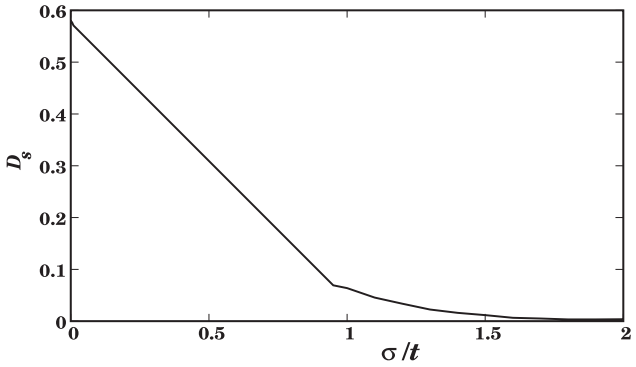

Figure 6. The superfluid stiffness, $D_{\rm s}$ is shown as a function of disorder. At large disorder, $D_{\rm s}$ becomes very small, implying a sharp decrease in superfluid density, $n_{\rm s}$.

pure system, however, the two processes happen at different temperatures in disordered systems. Another important quantity which provides an estimate of how superconductivity is affected in the presence of disorder is the spectral gap, $E_{\rm gap}$, obtained as the lowest eigenvalue of the BdG theory. $E_{\rm gap}$ and $\Delta_{\rm OP}$ are the same in the absence of disorder ($\sigma = 0$). However, the two show progressively different behaviour as a function of disorder, a feature which pushes the inhomogeneous BdG state to the unconventional regime (figure 5). The non-monotonic dependence of $E_{\rm gap}$ (bending upwards at larger disorder strengths) has been addressed via pairing of exact (noninteracting) eigenstates [39].

Next we focus on the superfluid stiffness $D_{\rm s}$ given by the Kubo formula [45]

$$\frac{D_{\rm s}}{\pi} = \langle -K_x \rangle - \Lambda_{xx}(q_x = 0, q_y \to 0, {\rm i}\omega = 0) \qquad (8)$$

where the first term $\langle -K_x \rangle$, defined as

$$\langle -K_x \rangle = \left\langle \sum_{\sigma} - \left[(t + \delta t_{i,i+\hat{x}})(c^{\dagger}_{i+\hat{x}} c_i + c^{\dagger}_i c_{i+\hat{x}}) \right] \right\rangle \qquad (9)$$

is the kinetic energy along the x-direction and represents the diamagnetic response to an external magnetic field. The paramagnetic response is given by the second term which is the disorder averaged transverse current–current correlation at different times and is given by

$$\Lambda_{xx}(\mathbf{q}, {\rm i}\omega_n) = \sum_{\mathbf{r}_i} \int_0^{\beta} {\rm d}\tau \langle j_x(\mathbf{r}_i, \tau) j_x(0,0) \rangle {\rm e}^{{\rm i}\mathbf{q}\cdot\mathbf{r}} {\rm e}^{{\rm i}-\omega_n \tau}. \quad (10)$$

It may be noted that the superfluid stiffness is large for pure systems where more energy is required to destroy phase rigidity, while it is less in the presence of disorder, which may be attributed to the formation of localized superconducting correlations (superconducting '*droplets*') in the inhomogeneous state. $D_{\rm s}$ is also looked upon as a measure of density of superelectrons, namely $n_{\rm s}$ ($D_{\rm s}/\pi = n_{\rm s}/m^*$, m^* being the effective mass). Figure 6 shows a considerable decrease in $D_{\rm s}$ (or equivalently $n_{\rm s}$, as m^* is negligibly affected) as σ is increased, a feature noted for unconventional superconductors such as the cuprates.

The unconventionality in the context of BCS–BEC crossover, i.e. the need to go beyond standard mean field theory (BCS) in the strong coupling limit is emphasized

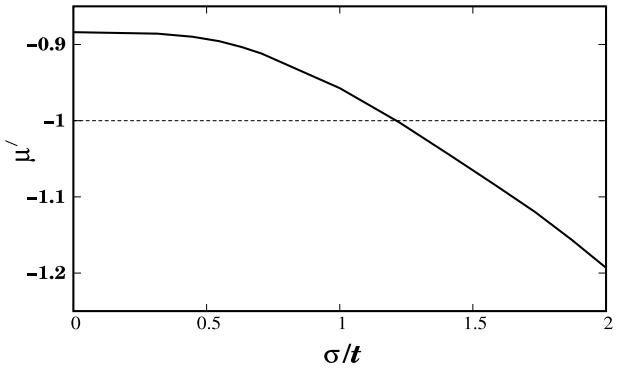

Figure 7. The scaled chemical potential, μ' is plotted as a function of disorder. μ' crosses the band minimum $\mu' = -1$ (shown by the dashed line) at $\sigma/t \simeq 1.2$.

Table 1. The contribution of the bound pairs (bosons) n_B to the number equation in BCS theory is shown at small and large values of disorder. The contribution is negligible at small disorder, but is significant in the presence of strong disorder.

Disorder strength (σ/t)	$n_B = \frac{\partial\langle H_{eff}(\sigma)\rangle}{\partial\mu}$
0.00	0.0000
0.01	0.0081
2.00	0.0146
2.50	0.0245
3.00	0.0323

by NSR [3]. The fluctuations due to the presence of bound pairs (bosons) are usually computed within a Gaussian approximation. The number equation incorporates the effect of Gaussian fluctuations, while leaving the gap equation unaltered [7]. At $T = 0$, the modification to the fermionic number density n appears in the form,

$$n' = n + n_B \tag{11}$$

where $n_B = -\frac{\partial\Omega_B}{\partial\mu}$ [30], Ω_B being the thermodynamic potential for the bosons.

The purpose of the above discussion is as follows—in a disorder driven crossover scenario, the system must consist of bound pairs in the large disorder regime. So the correction, i.e. the second term in equation (11) is inevitably present due to the formation of bound pairs. A rough estimate of this term, i.e. n_B or Ω_B can be done using[1],

$$\frac{\partial\Omega_B}{\partial\mu} \simeq \frac{\partial E}{\partial\mu} \sim \frac{\partial\langle H_{eff}(\sigma)\rangle}{\partial\mu} \tag{12}$$

where E represents the internal energy of the system and $\langle H_{eff}\rangle$ (defined by equation (4)) is computed within the BdG states. The results at small and large values of disorder presented in table 1 show that the correction term is negligible at small disorder and grows by roughly one order of magnitude in the presence of strong disorder.

[1] The thermodynamic potential $\Omega = -PV = KE$, where K is a constant which depends on statistics and dimensionality. See [46].

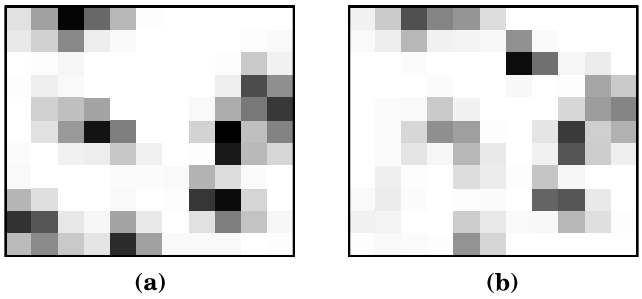

Figure 8. The greyscale plot of spatial charge distribution is shown for two different hopping disorder strengths, namely $\sigma/t = 1$ and 3. Unlike the onsite disorder case, the distribution is scattered at large disorder.

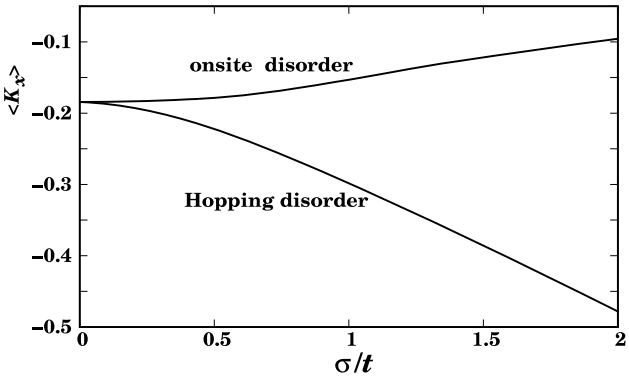

Figure 9. Average kinetic energy (negative of the expression used in equation (9)) for both hopping and onsite disorder as a function of disorder strength, σ.

The calculations presented above are only approximate, however they illustrate the importance of (bosonic) fluctuations in the presence of strong disorder for a fermionic superfluid.

We finally show the (scaled) chemical potential μ' as a function of disorder in figure 7. μ' crosses below the noninteracting band edge at $\sigma \simeq 1.2t$, separating a BCS-like superconductor (at lower σ) and a BEC phase (higher σ). To investigate density effects, we noted that μ' does not slip below the band minimum for larger densities (not shown here).

3.2. Hopping disorder

We study the effect of random hopping (off-diagonal) disorder on a superconductor, as our next candidate in the discussion on the BCS–BEC crossover. The ability of hopping disorder to induce a crossover phenomenon is under suspicion. The reason is as follows—random hopping events of the electrons cause delocalization, rather than a localized charge distribution, as was the case for onsite disorder. This can be seen by computing $\langle n_i\rangle$ for $\sigma/t = 1$ and 3 as shown in figure 8. To illustrate the presence of delocalization effects as contrary to localization caused by onsite disorder, we present the variation of average kinetic energy along the x-axis i.e. $\langle K_x\rangle$ for both kinds of disorder in figure 9. Further evidence of the contradictory behaviour that affects the crossover phenomena for these two kinds of disorder are presented in figure 10. Figure 10(a) shows

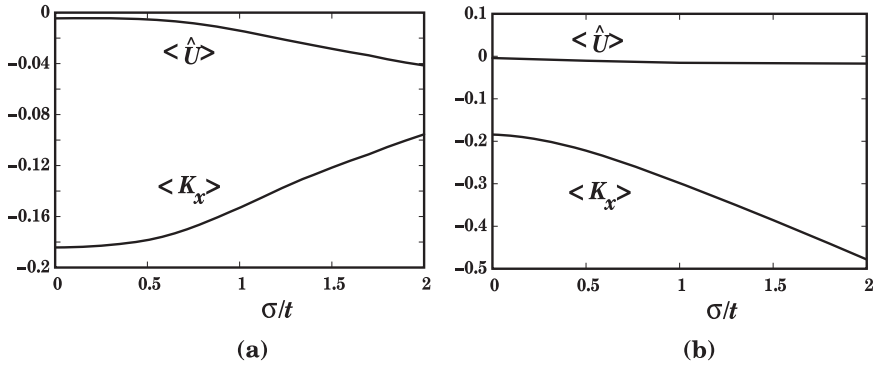

Figure 10. Average potential (defined in text) and kinetic energies are plotted as a function of disorder, σ for (a) onsite and (b) hopping disorder.

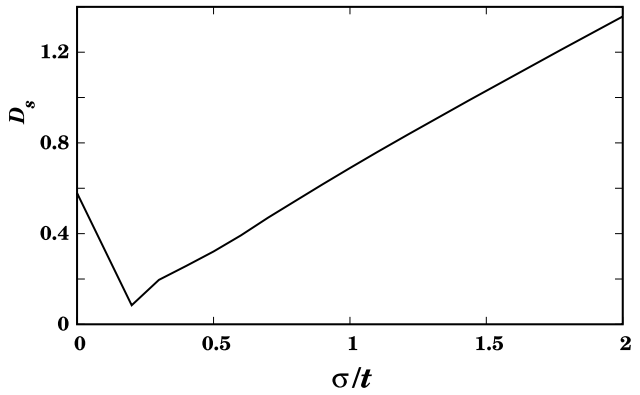

Figure 11. D_s is shown versus disorder strength for hopping disorder. It shows a gradual increase because the diamagnetic contribution (average kinetic energy) dominates the behaviour of D_s.

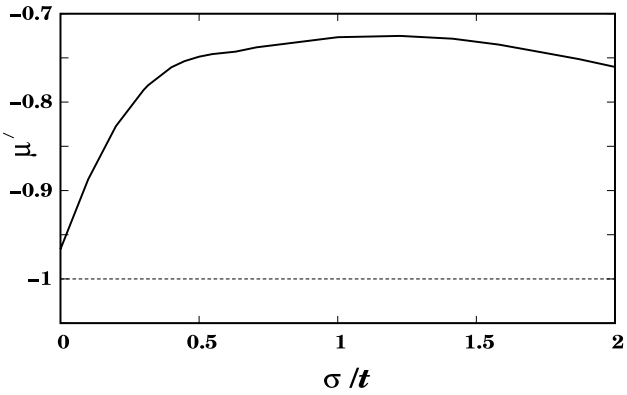

Figure 12. The scaled chemical potential is plotted as a function of σ for the case of hopping disorder. μ' does not cross the band edge, thereby ruling out a crossover scenario.

that for onsite disorder, the average potential energy, i.e. $\langle\hat{U}\rangle$ $(=|U|\langle\sum_i n_{i\uparrow}n_{i\downarrow}\rangle = |U|n_d)$ increases, while $\langle K_x\rangle$ decreases, thus both contributing to localization. The scenario is reversed for hopping disorder, where the sharp increase in kinetic energy renders delocalization, suppressing the negligible increase in average potential energy.

Also superfluid stiffness (figure 11) behaves differently than the onsite disorder case, i.e. D_s is found to increase with disorder. The explanation for this behaviour is contained in equation (8), where the first term on the left ($\langle -K_x\rangle$) dominates the behaviour of D_s in the presence of disorder and at large disorder D_s is entirely governed by this term. However, D_s falls initially at smaller values of disorder where an increase in the paramagnetic response offsets the rise in the diamagnetic term (see equation (8)). Expectedly, μ' in[2] figure 12 does not slip below the band edge and hence does not qualify to be a candidate for the crossover scenario. The enhancement of the noninteracting bandwidth owing to hopping disorder accommodates the chemical potential inside the band for all values of disorder.

We consider another situation where the carriers are allowed to move preferentially in one particular direction (say the x-direction), while the movement is strongly restricted in the other direction (y-direction) owing to anisotropic hopping frequencies. This is implemented in our calculations by introducing a tunable (anisotropy) parameter that gives a measure of the preferential hopping of the charge carriers. A similar problem was discussed earlier by us [43] in the context of BCS–BEC crossover, where an anisotropic hopping was considered in the strong coupling limit of the (repulsive) Hubbard, i.e. the t–J, model. We indeed found a crossover to exist at low densities and in the large anisotropy limit. The procedure was to solve BCS gap equations with a mixed symmetry for the superconducting gap parameter. The mixing (between (extended) s and d-wave channels) occurs due to lowering of the underlying lattice symmetry, i.e. from square to rectangular, in the presence of hopping anisotropies.

The crossover scenario is understood better with a reference to the dilute case. For two electrons, in the presence of hopping anisotropy, it was shown earlier that a bound state is possible with infinitesimal attraction in the limit of extreme anisotropy [42]. Moreover, it was found that a bound state becomes favourable for two electrons moving along different chains of a two-leg ladder (rather than along the chain) and hence form a stable pair when anisotropy is large. The same problem of confinement of the carriers can be investigated in a model where the isotropic (onsite) s-wave

[2] The scale factor $(=\mu(\sigma)/\mu'(\sigma))$ is obtained from a linear fit of noninteracting band minimum values as a function of σ, so as to keep the band minimum at $\mu' = -1$ for all values of σ.

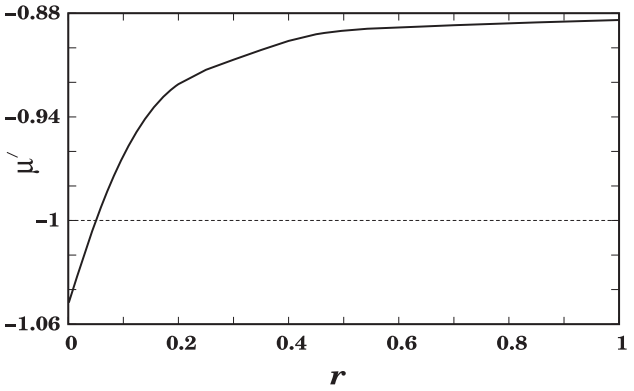

Figure 13. μ' is plotted versus anisotropy parameter $r\,(=\frac{t_y}{t_x})$ showing a crossing below the band minimum at very small values of r (the extreme anisotropy limit).

pairing correlations are strongest and also extend the two-particle problem to finite densities. The formalism that we have used in this paper, i.e. BdG approximations, provides scope to explore the scenario in a negative-U Hubbard model. Without bothering much about the physical properties of the anisotropic superconductor (we have computed the length scales that characterize the condensate earlier [43]), here we take a look at the chemical potential. The (scaled) chemical potential μ' $(=\mu/2t(1+r))$ as a function of the anisotropy parameter r is plotted in figure 13. μ' slips down the band minimum at a small value of r, i.e. in the extreme anisotropy limit, following band narrowing effects and thus presents a case for the crossover scenario [43].

4. Conclusions

We have investigated the effect of random disorder on the evolution of a BCS to a BEC superfluid using the BdG approximations on a two-dimensional square lattice. The study includes both onsite (diagonal) and hopping (off-diagonal) disorder. While onsite disorder presents a case for inducing a crossover due to localization effects, hopping disorder is shown to be *not* a catalyst for the crossover phenomenon, owing to delocalization of the charge carriers. The existence (or non-existence) of the crossover scenario is confirmed by calculating the chemical potential, which when it slips below the noninteracting band minimum, yields a crossover to a Bose phase. A third candidate of the crossover story, presented by the hopping anisotropy, yields a crossover in the extreme anisotropy limit, a possible artefact of the dimensional confinement of the charge carriers. The effect of carrier concentration on the crossover phenomena is also discussed and it is concluded that low density facilitates a crossover phenomena at moderate values of interparticle attraction. At higher densities, the overlap between the pairs increases substantially, thus denying access to a phase comprised of local pairs, reminiscent of a BEC phase.

We have presented mean field results, thus missing out the phase fluctuations which must be present in the strong disorder regime and may be important in the context of BCS–BEC crossover. A rough estimate of the fluctuations arising due to formation of short (and tightly bound) pairs is made which shows that the number equation (in BCS theory) should invoke the bosonic contribution in the presence of strong (onsite) disorder.

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